Fundamentals of medical imaging registration I - II

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What is registration?

- The process of aligning a target image to a source image
- More generally, determining the transform that maps points in the target image to points in the source image

\[
X' = T(X)
\]
• Why doing registration?
• What type of transformation?
• What type of similarity?
• How to estimate the transformation?
  – Geometric registration
  – Iconic registration
• How to resample the image?
• Why doing registration?
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Why doing registration?

Analysis of temporal evolution
Fusion of multimodal images
Inter-patients comparison
Atlas superposition
Reconstruction of a 3D volume
Temporal Evolution

Fusion of multimodal images

MRI

PET
(Positron emission tomography)

CAT

US

anatomical

Visible Man
Inter-patients comparison

\[ W_i = \frac{1}{N} \sum_{n=1}^{M} \vec{D}_n^T \vec{D}_n \]

Statistics on \( T_i \)

Compute brain variability
- Alzheimer's
- HIV/AIDS
- Schizophrenia
- Drug Abuse
- Development


Reconstruction of a 3D volume

- Series of contiguous 2D slices (thickness ~60nm) from Electron Microscopy camera
- The aim: to build a 3D volume

Different Classes of Problems

- Images: Mono or Multimodalities
- Intra- or Inter-subjects registration
- Transformation: Rigid or Non Rigid
Temporal Evolution

Time 1
- Sagittal
- Axial

Time 2
- Sagittal
- Axial

Intra Patient | Mono modality | Rigid & Non Rigid
Temporal Evolution

Preoperative MRI  Intraoperative MRI

Intra Patient  Monomodal  Non Rigid
Fusion of multimodal images

- MRI
- PET (Positron emission tomography)
- CAT
- US
- Visible Man
- anatomical

Intra Patient | Multi modal | Rigid
Compute brain variability
- Alzheimer's
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Inter Patient | Mono modal | Non Rigid

Atlas Superposition

Inter Patient | Multi modal | Non Rigid
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Intra Patient | Mono modality | Rigid & Non Rigid
Two Classes of solution

• Geometric Registration (or feature-based)
  – Extract feature points
  – Compute displacement of similar points
  – Fit a transformation with or without regularization

• Iconic Registration (or intensity-based)
  – Fit the transformation that optimizes similarity

Need:

• Define Transformation
• Define Similarity
• Define regularization
• Why doing registration?
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To resample: need for displacement field $T^{-1}(X)$

If $(X) = f(T^{-1}(X))$
How to resample the image?

Value $I(x,y)$?

$x' = \text{int}(x)$  \quad  $y' = \text{int}(y)$

- Nearest neighbor
  
  $I(x,y) = I(\text{round}(x), \text{round}(y))$

- Bilinear interpolation
  
  $a = x - x'$  \quad  $b = y - y'$

  \[
  I(x,y) = a(1-b)I(x'+1,y') + (1-a)(1-b)I(x',y') + abI(x'+1,y'+1) + b(1-a)I(x',y'+1)
  \]

- Bicubic, splines ....
• Resampling of a vector field:
  – \( X' = T(X) \) or \( U = F(X) \)
  – Compute local affine transformation (Jacobian):
    \[
    J = \nabla T
    \begin{pmatrix}
      \frac{\partial T_x}{\partial x} & \frac{T_x}{x} & \frac{T_x}{z} \\
      \frac{\partial T_y}{\partial y} & \frac{T_y}{y} & \frac{T_y}{z} \\
      \frac{\partial T_z}{\partial z} & \frac{T_z}{z} & \frac{T_z}{z}
    \end{pmatrix}_x
    = (\nabla T)_x
    \]
  – Update Vector:
    \[
    V_f'(X) = J(T^{-1}(X))V_f(T^{-1}(X))
    \]

• Tensors:
  Recall: a tensor can be written:
  \[
  W(X) = \sum_{i=1}^{3} \bar{V}(X)\bar{V}(X)^T
  \]
  Theoretically:
  \[
  W_f'(X) = J(T^{-1}(X))W_f(T^{-1}(X))J(T^{-1}(X))^T
  \]
  But … what does the tensor and the transformation mean?
• Don’t forget the Nyquist–Shannon theorem!
  
  To downsample the image by a factor N (each dimension)
  – Remove the high frequency of the image:
    • Fast Fourier Transform (FFT) ⇒ high freq = 0 ⇒ FFT\(^{-1}\)
    • Or Gaussian filter \(\sigma \sim N\)

  ![Diagram showing high frequency set to 0 through FFT and FFT\(^{-1}\) operations](image)
Overview

• Why doing registration?
• **What type of transformation?**
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Transformations Class $T$

- Rigid (displacement)
- Similitude
- Affine
- Polynomial
- Mesh-based
- Splines
- Free
• **Rigid:** $T(x) = Rx + t$
  - Rotation (R) and translation (t)
  - 6 parameters: (R : 3; t : 3)
  - Invariants: distances (isometric), orientation, curvature, angles, lines

• **Similitude:** $T(x) = s.Rx + t$
  - Adds a scaling factor
  - 7 parameters
  - Invariants: distance ratio, orientation, angles, line
Transformations Class T

- **Affine**: \( T(x) = Bx + t \)
  - B 3x3 matrix
  - 12 parameters: (B : 9; t : 3)
  - invariants: lines, parallelism

- **Polynomial**: \( T(x) = P(x) \)
  - Parameters: \( D \sum_{o=1}^{N} \Gamma_{D}^{O} = D \sum_{o=1}^{N} C_{D+O-1}^{O} \)
  - D = dimension, O = order (or degree)
  - Example, D=2, O=2

\[
P_x(x, y) = a_1 + b_1x + c_1y + d_1xx + e_1xy + f_1yy \\
P_y(x, y) = a_2 + b_2x + c_2y + d_2xx + e_2xy + f_2yy
\]
• **Mesh-based (piecewise affine):**

For $X \in T (P_1, P_2, P_3)$

$$T(X) = \sum_{i=1}^{3} h_i P_i$$

$$
\begin{bmatrix}
    h_1 \\
    h_2 \\
    h_3
\end{bmatrix} =
\begin{bmatrix}
    P_1^x & P_2^x & P_3^x \\
    P_1^y & P_2^y & P_3^y \\
    1 & 1 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix}
$$
• **Splines:**
  - Local polynomials $d$, with global continuity of order $C(d-1)$.
  - Number of parameters: depends on the number of nodes
  - Locally affine transformations

• **Free Transformation:** $T(x) = x + u(x)$
  - One displacement $u(x)$ per voxel
  - Parameters: 3 times number of voxels
  - Need for regularization to ensure diffeomorphism
• More Transformations …

– Cosine (SPM)

– Multi-affine

– Poly-affine

– Radial Basis Functions

– Wavelet
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What type of similarity?

- Depends on the joint histograms:

Image 1

Intensity = 80

Image 2

Intensity = 100
Similarity measures

• Assumption:
  Same intensity

• Adapted measure

Sum of squared difference

\[ S(T) = \sum_k (i_k - j_k)^2 \]

Sum of absolute differences

\[ S(T) = \sum_k |i_k - j_k| \]

Interpolation: \( j_k \Downarrow \equiv J(T(x_k)) \)
• Assumption:
  Linear relation

• Adapted measure

Correlation coefficient

$$\rho_{IJ}(T) = \sum_{k} \frac{(i_{k} - \bar{I})(j_{k} \downarrow - \bar{J})}{i_{k} j_{k} \downarrow - \bar{IJ}}$$
• Assumption:
  Statistical relationship

• Adapted measures

Joint Entropy (Hill, 95; Collignon, 95)
Mutual Information (Collignon, 95; Viola, 95)
Normalized Mutual Information (Studholme, 98)

\[
MI(I, J) = H(I) + H(J) - H(I, J)
\]

\[
= \sum_i \sum_j P(i, j) \log \frac{P(i, j)}{P(i)P(j)}
\]
• More measures:


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Simple example (1):
Simple example (1):

- Extract feature points (structure tensor)
  - \( T = G * (\nabla I)(\nabla I)^T \)
  - Select n largest \( T \)

T. Hartkens, K. Rohr, and H. S. Stiehl "Evaluation of 3D Operators for the Detection of Anatomical Point Landmarks in MR and CT Images" Computer Vision and Image Understanding, Volume 86, Number 2, pp 118-136, 2002
Simple example (1):
- Find local correspondences
  - Block matching
Find optimal transformation

$$\operatorname{argmin}_T \left( \sum_k \left\| T(x_k) - y_k \right\|^2 \right)$$
Simple example (2), Iterative Closest Point:

- Consider 2 sets of points \( I_1 = \{P_i\} \) and \( I_2 = \{Q_j\} \)
- Consider an initial estimate of the transformation \( T \) (usually identity)

1. Each point \( P_i \) of \( I_1 \) is paired with the closest point \( Q_j \) of \( I_2 \)
2. We look for \( T_i \) that minimizes the squared distance between impaired points
3. Update the position of \( P_{i+1} = T_i (P_i) \)

- Iteration until convergence
1. Optimization of Pairings

$Q_j$  $P_i$
2. Optimization of $T$
1-bis Optimization of Pairings

Diagram showing pairings with labels Qj and Pi.
2-bis. Optimization of $T$

(Qj, Pi)
• More general case:
  – Parametric Transformation $T$, parameters $p_i$
  – Regularization energy $E(T)$.
    • Mechanical energy $U K U^T$
    • Gradient based $\nabla U (\nabla U)^T$

  – Minimize the sum of the two:

$$\arg \min_{p_i} \left( E_r(T) + \sum_k \| T(x_k) - y_k \|^2 \right)$$
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Iconic registration

Basic idea: try to find the transformation parameters that directly minimize the similarity between the 2 images

$$\arg \min_{p_i} \left( E_r(T) + E_s(I_{F}^{\downarrow}, I_T) \right)$$

$$= UKU^T$$

$$= \sum_k (i_k - j_k^{\downarrow})^2$$

$$= \nabla U \left( \nabla U \right)^T$$

$$= \sum_k \frac{(i_k - \bar{I})(j_k^{\downarrow} - \bar{J})}{i_k j_k^{\downarrow} - \bar{IJ}}$$

$$= \sum_i \sum_j P(i, j) \log \frac{P(i, j)}{P(i) P(j)}$$
• How to optimize the functional?
  – Can you compute analytical solution?
  – Can you compute the derivative?
    • No: Powell, downhill simplex, genetic algorithms
    • Yes: gradient descent

More details: numerical recipes in C